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Cox-Based Estimation Model for Critical Illness Insurance Policy for Breast Cancer Based on the Possible Transition of Status

Sari, Y. W.[1], Gunardi*[1], Megawati, N. Y.[1], and Hutajulu, S. H.[2]

[1]*Department of Mathematics, Faculty of Mathematics and Natural Sciences, Universitas Gadjah Mada, Yogyakarta, Indonesia*
[2]*Department of Internal Medicine, Faculty of Medicine, Public Health and Nursing, Universitas Gadjah Mada, Dr. Sardjito General Hospital, Yogyakarta, Indonesia*

E-mail: gunardi@ugm.ac.id
**Corresponding author*

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Abstract

Breast cancer treatment involves complex medical procedures with high costs. These high costs highlight the need for adequate insurance coverage. In Indonesia, most cancer insurance schemes offer lump sum payouts upon diagnosis. In this study, we develop a cancer insurance scheme that provides benefits for patients who need extended treatment periods until they are declared cancer-free. This scheme is based on the possible transitions in the status of breast cancer patients, considering factors such as patient age, cancer stage and comorbidities. The time-homogeneous Cox Markov model is used to assess the impact of various covariates on patient status transitions and estimate the transition intensities between patient statuses. The proposed models and methods are then applied to data from patients with breast cancer at Dr. Sardjito Hospital, Yogyakarta, Indonesia. The application of the model to the data indicates that premiums for both stand-alone benefit and endowment benefit cancer insurance policies tend to be higher for older patients, those in advanced stages and those with hypertension. This developed model is a valuable method for insurance companies to estimate the probability of treatment status transitions and insurance premium rates in breast cancer patients, as well as to develop new insurance products that are critically needed by patients.

Keywords: breast cancer; Cox proportional hazards model; time-homogeneous model; critical illness insurance rates; multistate framework; oncology coverage.

1 Introduction

Cancer is one of the critical non-communicable diseases that has gained worldwide attention. Various types of cancer are characterized by their unique features, effects and different medical treatment approaches. Breast cancer is the most prevalent cancer in women and causes the second-highest rate of mortality. According to Globocan [14], there were $19,292,789$ reported cancer cases worldwide in 2020, with $2,261,419$ (11.7%) attributed to breast cancer.

Chemotherapy is one of the frequently recommended cancer treatment methods. After a patient is diagnosed with cancer, the doctor will create a treatment plan or chemotherapy schedule. During this treatment process, the patient's status may change; they may complete the treatment successfully or, unfortunately, they may pass away while the process is ongoing. The survival ability of cancer patients is influenced by the patient's age and the stage of the cancer [3, 7]. It is in part due to the functions of the organs of the human body, especially the liver and kidneys, which decline with age [26]. Furthermore, comorbidities significantly affect the initiation, delays and discontinuation of treatment [13]. The most common comorbidity experienced by cancer patients is hypertension [8, 23]. Hypertension will increase cardiovascular risk, leading to an increased risk of mortality in cancer patients [22]. Therefore, doctors always consider age, cancer stage and hypertension when planning cancer treatment.

Each cancer patient encounters dual challenges concurrently: the medical treatment itself and the associated treatment expenses, as highlighted in the study by [27]. The financial challenges of cancer treatment expenses underscore the importance of adequate insurance coverage for individuals and families. Some cancer insurance schemes provide a lump sum insured if the insured is diagnosed with cancer, regardless of the total treatment costs required until recovery. Other schemes provide benefits at cost within a specific period during the insurance contract. In fact, patients also need financial protection in the event that they need more time to complete treatment until they are declared cancer-free.

The multi-state model serves as a mathematical framework for representing systems that facilitate transitions among different states or conditions. Transition probabilities between states can be determined by integrating transition intensity estimates derived from the Poisson model and semi-parametric regression methods, such as Cox regression [4, 25]. In later studies, Andersen and Perme [5] noted that a notable feature of the multistate model is that all hazard-based models in survival analysis can be utilized as transition intensities. Prominent hazard models include the Cox model, the Gompertz model, the Gompertz-Makeham model and their extensions, such as the hazard rate model with the Gompertz Flexible Weibull distribution developed by [17]. Among these various hazard models, the Cox model is particularly notable, as it allows the inclusion of covariates that play a significant role in state transitions. In the Cox model, baseline hazard estimation can be performed using the maximum partial log-likelihood method [18] or the Taylor series approximation [2]. Various assumptions regarding the time-dependent nature of transition intensities can be made, including homogeneous models, Markov models and semi-Markov models [21].

Changes in cancer patient status are typically managed through multi-state models. Multi-state models represent the evolution of patient status, similar to a sample path of a continuous-time Markov chain, as observed in studies by [21, 12]. These multiple states correspond to distinct phases of a patient's journey throughout the cancer treatment process. The application of multistate models proves suitable for analyzing the time elapsed until a specific event transpires, as discussed in the work by [20] and references to the Weibull distribution [29]. Compared to standard survival methods, the multi-state model offers more in-depth information about the effects

of treatment in cancer clinical trials [19]. Consequently, the multistate model is a pertinent and valuable modeling framework to understand cancer development and treatment dynamics [7, 4].

Several studies have been conducted to calculate critical illness insurance premiums considering changes in patient status. Baione and Levantesi [6] employed a multi-state model to estimate critical illness insurance premiums, basing the transition probabilities between patient states on prevalence rates. In 2019, Pasaribu et al. [24] used a multiple-state continuous Markov chain model to calculate critical illness insurance premiums, with transition intensities derived from CMI Working Paper 50. A discrete-time approach in a multistate non-homogeneous Markov chain model is another method applicable for modeling critical illness insurance [9]. Similarly, Taraly et al. [28] conducted a study where transition intensities between patient states were estimated using the 2019 Indonesian Mortality Table. The Gompertz-Makeham model and the Bayesian model are additional approaches utilized for estimating transition intensities [1, 16]. Fathoni et al. [11] calculated the transition probabilities between patient states (cancer-free, early stage, advanced stage and deceased) by solving the Kolmogorov forward differential equation. These transition probabilities were then used to compute critical illness insurance premiums for breast cancer.

Driven by the need for a comprehensive yet easily interpretable model capable of assessing status transitions in breast cancer patients during treatment, while considering factors such as age, disease severity and comorbidities, this study utilizes the Cox Proportional Hazards model within a multi-state framework. This approach not only facilitates straightforward interpretation but also eliminates the need for specific distributional assumptions and efficiently handles censored data. Furthermore, the multi-state model is applied to the modeling of critical illness insurance for breast cancer. While previous research has employed the Cox model within multi-state frameworks, it has not yet been used in insurance modeling. Additionally, unlike traditional critical illness insurance plans that offer benefits upon breast cancer diagnosis, this study proposes an insurance scheme in which benefits are paid if the treatment process takes longer than initially planned. In addition, this model offers valuable insights that can improve decision-making across various clinical and financial aspects, particularly in relation to breast cancer treatment and insurance planning.

In Section 2, we provide a model construction and the structure of multi-state models. Section 3 outline the methodology for pricing insurance. Moving on to Section 4, we present a case study focused on breast cancer patients from the Dr. Sardjito Hospital in Yogyakarta, where the proposed models and methods are applied. The concluding insights and discussions are presented in Section 5. This organization ensures a systematic exploration of the proposed models, their application to real-world data and a conclusive discussion of the findings.

2 Model Construction

The treatment journey for breast cancer patients invariably involves a sequence of chemotherapy sessions and the duration, frequency and overall length of these cycles vary for each individual. The severity or stage of cancer significantly influences the disparities in these aspects. In the context of this study, the observation period for breast cancer patients spans a one-year observation period. Some patients complete their chemotherapy, while others also need a longer time frame for their treatment. Unfortunately, some patients do not survive until the course of chemotherapy is completed. The pivotal factors that influence the survival rates of cancer patients are age, cancer stage and comorbidity [3, 7].

Let $\{S(t)\}_{t\in[0,T]}$ be a stochastic process defined in continuous time intervals $[0,T]$ and $S = \{1,2,3\}$ is a finite state space. Using a multiple-state model, the transitions between states depict distinct phases in the breast cancer journey. State-1 signifies the status of a patient who has been diagnosed and undergoing chemotherapy, State-2 indicates the status of successfully completing a course of chemotherapy and State-3 represents the status of a deceased patient. This modeling approach enriches our understanding of the varied trajectories breast cancer patients can experience, contributing to a more nuanced understanding of their medical progression.

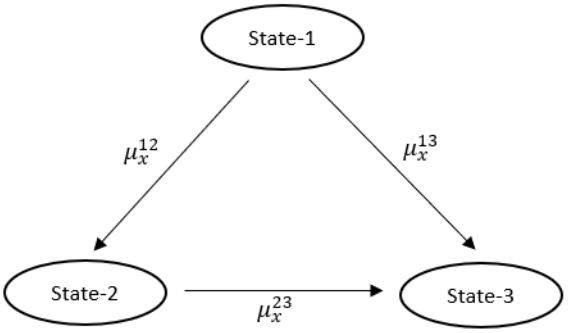

Figure 1: The transition scheme between states.

Using the previously defined state space, we illustrated the transitions between states as depicted in Figure 1. Transition 1 signifies the progression of a breast cancer patient from the diagnosis and chemotherapy phase to the completion of chemotherapy. Transition 2 illustrates the shift in a patient's status from being diagnosed and undergoing chemotherapy to the unfortunate event of dying before completing the chemotherapy. Transition 3 captures the alteration in patient status from complete treatment to succumbing to the disease. It is important to note that our focus was solely on changes in patient status within a single therapy cycle. Hence, we excluded the consideration of patients who transition from completed chemotherapy status to chemotherapy for a second cycle.

The transition probability, denoted as ${}_tP_x^{ij}$, represents the likelihood that an insured individual aged x will transition from state-i to state-j within the time interval t. The transition probability is calculated using the formula,

$$ {}_tP_x^{\bar{i}i} = exp\left[-\int_0^t \sum_{i\neq j} \mu_{x+u}^{ij} du\right], \tag{1}$$

$$ {}_tP_x^{ij} = \int_0^t {}_uP_x^{\bar{i}i} \mu_{x+ut-u}^{ij} P_{x+u}^{\bar{j}j} du, \tag{2}$$

where μ_{x+t}^{ij} represents the transition intensities or rates and u is the time interval $(0 < u < t)$. The transition rate reflects the instantaneous failure rate of moving from state-i to state-j, analogous to the hazard function in survival analysis [15].

In this study, the Cox regression model is used to estimate the transition intensities. Let the number of samples be n, then the transition intensity estimator of an individual h, where

$h = 1, 2, \ldots n$, can be written as,

$$\hat{\mu}_{x+t}^{ijh} = h_{ij}(t|Z_h) = h_{ij0}(t)exp\left(\sum_k \beta_{ijk} Z_{ijkh}\right),$$

where for each transition from state-i to state-j, $h_{ij0}(t)$ is the baseline hazard function, β_{ijk} is the Cox regression coefficient for the k-th covariate, which is assumed to be constant and Z_{ijkh} is the value of k-th covariate of individual h.

The Cox regression coefficients of transition from state-i to state-j are estimated using maximum partial log-likelihood estimation method. Furthermore, if the event is defined as the transition from state-i to state-j and a sequence of event times exists, then t_h represents the time of the h-th event. The cumulative baseline hazard rate is given by $H_{ij0}(t) = \int_0^t h_{ij0}(s)ds$ and the estimation of $H_{ij0}(t)$ is

$$\hat{H}_{ij0}(t) = \sum_{t_h \leq t} \frac{N_{ijt_h}}{\sum_{l_{ij} \in R_{t_h}} \exp\left(\sum_k \beta_{ijk} Z_{ijkl_{ij}}\right)},$$

where N_{ijt_h} is the number of individuals who transitioning from state-i to state-j at t_h, l_{ij} is the individual who tend to get the event and R_{t_h} is the set of individuals who have the risk of getting the event at time t_h.

The multi-state Markov model can be divided into two types based on the assumption of transition intensity functions over time: the time homogeneous Markov model and the non-homogeneous model. In the time homogeneous Markov model, all transition intensities are assumed to be constant over time. In contrast, the non-homogeneous model assumes that transition intensities change over time. The non-homogeneous Markov process can be modeled non-parametrically, for example, using the Kaplan-Meier approach [21].

3 Insurance Rates Model

Cancer insurance is a type of critical illness insurance that protects against financial risks due to cancer. In cancer insurance, the insured will receive benefits if diagnosed with cancer or if they experience certain conditions related to cancer. Additionally, like other critical illness insurances, cancer insurance typically includes death benefits. We define a formula to calculate the pure premium for an N-year cancer insurance policy based on the previously discussed multi-state model. This premium is calculated under the assumption of a time-homogeneous Cox Markov model.

In this study, the critical illness insurance model for breast cancer assumed that pure premiums are paid once (a single pure premium) when the contract is signed. Therefore, the premium rate can be seen as the expected present value (EPV) by the unit of insurance benefits. In this study, we discuss two insurance models based on the type of insurance benefits: stand-alone benefit and endowment benefit.

3.1 Stand-alone benefit

A stand-alone benefit in breast cancer insurance refers to a separate insurance policy that covers specific risks related to cancer that are not included in an existing insurance policy. In this study,

the insurance policy provides individual protection for the risk of remaining in state-1 without any connection to other insurance products.

If the insured is still alive and has not yet completed the series of chemotherapy until the end of the contract, the insured will receive B_1. As a result, the premium of the N-year stand-alone benefit (SA) is the discounted value of B_1, assuming he remains in status-1 until N-year later. As defined in the transition scheme for breast cancer patients in Figure 1, the premium of the N-year stand-alone benefit is expressed as follows,

$$\bar{A}^{(SA)}_{x:N} = B_1 {}_N P^{\bar{11}}_x v^N, \tag{3}$$

where

$$\begin{aligned} {}_N P^{\bar{11}}_x &= \exp\left(-\int_0^N \mu^{12}_{x+u} + \mu^{13}_{x+u} du\right) \\ &= \exp\left(-h_{120}\exp\left(\sum_k \beta_{12k} Z_{12k}\right) N - h_{130}\exp\left(\sum_k \beta_{13k} Z_{13k}\right) N\right), \end{aligned}$$

expresses the probability of an insured aged x being still alive and not having completed chemotherapy at time N. In addition, v is the discount factor or $v = \dfrac{1}{(1+r)}$, where r is the interest rate. If the r is continuously compounded rate of interest, then $v^N = \exp(-rN)$ [10, 30].

3.2 Endowment benefit

The endowment insurance policy provides a combination of stand-alone benefits with an additional death benefit. The model assumes that the benefits in this breast cancer insurance model are provided if the insured dies or has not completed the chemotherapy course at the maturity time of the insurance contract. Therefore, the premium of the endowment benefit (EB) is the sum of the premium of death benefit and the premium of stand-alone benefit described in the previous sub-section.

The death benefit is paid if the insured dies during the insurance contract, say N years. If the policy is continuous, the benefit is paid immediately upon the insured's death. It is assumed that if the insured dies before or after completing chemotherapy, they will receive B_2. According to [10], the premium for an N-year death benefit (DE) for an individual currently aged x is defined as,

$$\bar{A}^{(DE)}_{x:N} = B_2 \left[\int_0^N {}_t P^{\bar{11}}_x \mu^{13}_{x+t} v^t dt + \int_0^N {}_t P^{12}_x \mu^{23}_{x+t} v^t dt\right]. \tag{4}$$

Equation (4) can be explained as the sum of the premium of insured who died before or after completing the chemotherapy course. The present value of the expectation that the insured aged x will die before completing chemotherapy between ages x and $x+N$ is defined as,

$$\begin{aligned} &\int_0^N {}_t P^{\bar{11}}_x \mu^{13}_{x+t} v^t dt \\ &= \frac{h_{130}\exp\left(\sum_k \beta_{13k} Z_{13k}\right)}{h_{120}\exp\left(\sum_k \beta_{12k} Z_{12k}\right) + h_{130}\exp\left(\sum_k \beta_{13k} Z_{13k}\right) + r} \\ &\quad \times \left[1 - \exp\left(-N\left(h_{120}\exp\left(\sum_k \beta_{12k} Z_{12k}\right) + h_{130}\exp\left(\sum_k \beta_{13k} Z_{13k}\right) + r\right)\right)\right]. \end{aligned}$$

Meanwhile, the present value of the expectation that the insured aged x will die after completing chemotherapy between ages x and $x+N$ is expressed by,

$$\int_0^N {}_tP_x^{\bar{12}}\mu_{x+t}^{23}v^t dt = \frac{ac}{(a+b-c)}\left[\frac{1-\exp(-(c+r)N)}{c+r} - \frac{1-\exp(-(a+b+r)N)}{a+b+r}\right],$$

where

$$a = \mu_{x+t}^{12} = h_{120}\exp\left(\sum_k \beta_{12k}Z_{12k}\right),$$

$$b = \mu_{x+t}^{13} = h_{130}\exp\left(\sum_k \beta_{13k}Z_{13k}\right),$$

$$c = \mu_{x+t}^{23} = h_{230}\exp\left(\sum_k \beta_{23k}Z_{23k}\right).$$

Furthermore, the premium for the endowment insurance policy is calculated by,

$$\bar{A}_{x:N}^{(EB)} = \bar{A}_{x:N}^{(DE)} + \bar{A}_{x:N}^{(SA)}. \tag{5}$$

4 Application and Results

In this section, we applied the multi-state model to data from breast cancer patients at Dr. Sardjito Hospital in Yogyakarta. A total of one hundred sixty-eight patients in the Integrated Cancer Installation "Tulip" at Dr. Sardjito Hospital were observed from July 2018 to June 2020. The data were obtained from [11]. Based on these data, the probabilities that a breast cancer patient aged x will die, complete therapy, or still undergo therapy at age $x+1$ were calculated. The EPV was also calculated to determine the cancer insurance premium rate. In its application, the software R is used and some of the packages utilized include survival and mstate.

According to medical research, cancer stages, age and comorbidities significantly affect the survival of breast cancer patients [3, 23]. Hypertension is the most common comorbidity among breast cancer patients [8, 23]. Hypertension is considered by doctors in the treatment of breast cancer because it can cause cardiovascular complications. As a result, cancer stages, age and hypertension status are used as independent variables (covariates) to estimate transition intensities.

Table 1: The covariates.

Covariate	**Patient grouping**
Cancer stage	1. early and intermediate stage
	2. advanced stage
Ages	1. < 50 years
	2. ≥ 50 years
Comorbidity	1. without hypertension
	2. hypertension

Table 1 contains information about these variables. We categorized breast cancer patients into two groups based on their cancer stage: early-stage and intermediate-stage (stages I, II and III)

and advanced-stage (stage IV). We combined early-stage and intermediate-stage cancer patients into a single group because, based on the data we used, there was no significant difference in survival between the two stages.

Furthermore, Table 2 presents the estimated coefficients for each covariate at each transition. The state transition of breast cancer patients is defined in Figure 1.

Table 2: The estimation of coefficient of covariates.

Covariate	**Transition**	**Notation**	**Coefficient**	$p-$**value**
Cancer stage	1	$\hat{\beta}_{121}$	-0.5593	0.00782
	2	$\hat{\beta}_{131}$	1.9657	0.00259
	3	$\hat{\beta}_{231}$	2.3486	0.00014
Age	1	$\hat{\beta}_{122}$	-0.3576	0.05335
	2	$\hat{\beta}_{132}$	-1.2056	0.02240
	3	$\hat{\beta}_{232}$	0.2494	0.65843
Comorbidity	1	$\hat{\beta}_{123}$	-0.3924	0.06378
	2	$\hat{\beta}_{133}$	0.6690	0.18982
	3	$\hat{\beta}_{233}$	0.1320	0.83212

As indicated in Table 2, analysis of the breast cancer patient data from Dr. Sardjito Hospital reveals that the $p-$values for cancer stage across all transitions, age between transitions 1 to 2 and 1 to 3 and hypertension between transition 1 to 2 are all below 0.1. This suggests that the stage of cancer has a significant influence on all status transitions among breast cancer patients at the 10% significance level. In contrast, age only significantly influences the transitions from status 1 to 2 and from status 1 to 3, whereas hypertension has a significant effect solely on the transition from status 1 to 2.

For subsequent analysis, only those covariates demonstrating significant effects are considered. The findings from the proportional hazards test, as shown in Table 3, reveal that the $p-$values for all covariates associated with the three patient status transitions exceed 0.05. This suggests that, at the 5% significance level, the relationship between each covariate and the time to event adheres to the proportional hazards assumption.

Table 3: The result of proportional hazard test.

Covariate	**Transition**	**Notation**	**Chisq**	$p-$**value**
Cancer stage	1	$\hat{\beta}_{121}$	3.07	0.080
	2	$\hat{\beta}_{131}$	1.79	0.181
	3	$\hat{\beta}_{231}$	1.34	0.248
Age	1	$\hat{\beta}_{122}$	2.73	0.098
	2	$\hat{\beta}_{132}$	0.36	0.547
Comorbidity	1	$\hat{\beta}_{123}$	1.86	0.170

The cumulative baseline hazard plots for the three transitions in the status of breast cancer

patients exhibit a near-linear pattern (see Figure 2). This suggests that the time-homogeneous model assumption (constant baseline hazard over time) can be applied to this dataset.

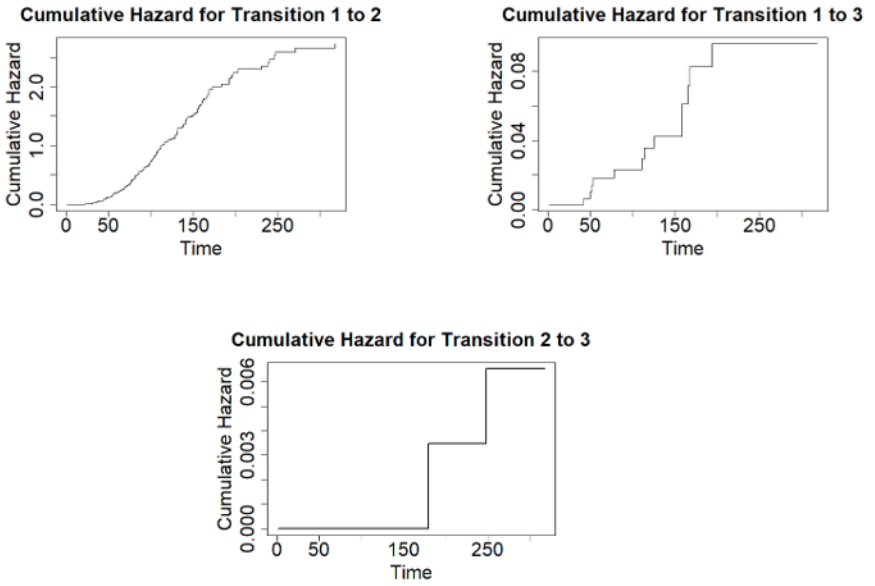

Figure 2: Cumulative baseline hazard of the Cox proportional hazard model.

Table 4 illustrates the probability of breast cancer patients transitioning within one year for each age group, cancer stage and hypertension status. The transition probability of patients who die after completing a chemotherapy cycle is influenced only by the cancer stage (Table 5). Patients under 50 years of age with early and intermediate stage cancer are more likely to complete chemotherapy than older patients with more advanced cancer. In the transition from state-1 to state-3, the higher the cancer stage, the greater the probability of dying before completing chemotherapy. However, Table 4 shows that this probability decreases with increasing age. Generally, breast cancer patients with hypertension had a higher chance of dying within a year than patients without hypertension. In addition, people without hypertension have a higher chance of completing chemotherapy.

Table 4: The transition probabilities ${}_1P_x^{ij}$.

Status transition	Age interval	Cancer stage	
		Early and intermediate stage	Advanced stage
$1 \to 1$	Hypertension		
	< 50	0.14348	0.17349
	≥ 50	0.26624	0.37755
	Without hypertension		
	< 50	0.05910	0.10448
	≥ 50	0.14317	0.26483
$1 \to 2$	Hypertension		
	< 50	0.80949	0.46960
	≥ 50	0.71182	0.44722
	Without hypertension		
	< 50	0.90321	0.58231
	≥ 50	0.83753	0.57185
$1 \to 3$	Hypertension		
	< 50	0.04219	0.32862
	≥ 50	0.01798	0.15088
	Wihout hypertension		
	< 50	0.03181	0.27613
	≥ 50	0.01430	0.13064

Table 5: The transition probabilities ${}_1P_x^{ij}$.

Status transition	Cancer stage	
	Early and intermediate stage	Advanced stage
$2 \to 2$	0.99090	0.91209
$2 \to 3$	0.00910	0.08791

Next, the expected present value per unit benefit (premium rate) of the stand-alone benefit and endowment benefit are calculated for a 1-year contract period. In this case study, the discount interest rate is 5.75%. The interest rate is the reference interest rate for monetary policy in Indonesia, set by Bank Indonesia and called the BI rate. Tables 6–7 illustrates the rates of pure single insurance premium in the event of specific risks. Furthermore, the amount of insurance premium is calculated by multiplying the premium rate by the benefit units obtained within one year. In this study, the stand-alone and death benefit units are denoted as B_1 and B_2, respectively.

Table 6 illustrates the insurance premium rate that cancer patients must pay to receive a benefit of B_1 if the chemotherapy course is not completed by the end of the contract. For example, the premium rate for early-stage hypertension patients under 50 years old is 0.13546. This means that the premium the insured must pay is 13.546% of B_1. As compensation, the patient will receive an

insurance benefit of B_1 at the end of the contract if chemotherapy has not been completed. Premiums for patients without hypertension are lower than for those with hypertension. Additionally, patients with a higher cancer stage have to pay a more expensive premium. The same applies to patients who are 50 years old or older.

Table 6: The premium rate of stand-alone benefit policy.

Age interval	**Cancer stage**	
	Early and intermediate stage	**Advanced stage**
Hypertension		
< 50	0.13546	0.16379
≥ 50	0.25136	0.35645
Without hypertension		
< 50	0.05579	0.09865
≥ 50	0.13517	0.25003

Under the assumption $B_1 = B_2 = B$, based on Table 7, early-stage or intermediate-stage breast cancer patient under 50 years with hypertension must pay 18.150% of B to receive B if they die or B if they have not completed chemotherapy within one year of diagnosis. Table 7 also shows that the premium increases with age and cancer stage. The insurance premium rate of breast cancer patients with hypertension are higher than patients without hypertension.

Table 7: The premium rate of endowment benefit policy.

Age interval	**Cancer stage**	
	Early and intermediate stage	**Advanced stage**
Hypertension		
< 50	0.18150	0.51300
≥ 50	0.27276	0.52723
Without hypertension		
< 50	0.09276	0.40548
≥ 50	0.15402	0.40930

Furthermore, Figures 3–6 demonstrate how variations in insurance contract length (N) and baseline hazard (h_{ij0}) impact the performance of the breast cancer insurance premium calculation model. The covariate combinations for each policy are denoted by letters as follows:

a: Early and intermediate stage, age < 50, with hypertension.

b: Early and intermediate stage, age ≥ 50, with hypertension.

c: Early and intermediate stage, age < 50, without hypertension.

d: Early and intermediate stage, age ≥ 50, without hypertension.

e: Advanced stage, age < 50, with hypertension.

f: Advanced stage, age ≥ 50, with hypertension.

g: Advanced stage, age < 50, without hypertension.

h: Advanced stage, age ≥ 50, without hypertension.

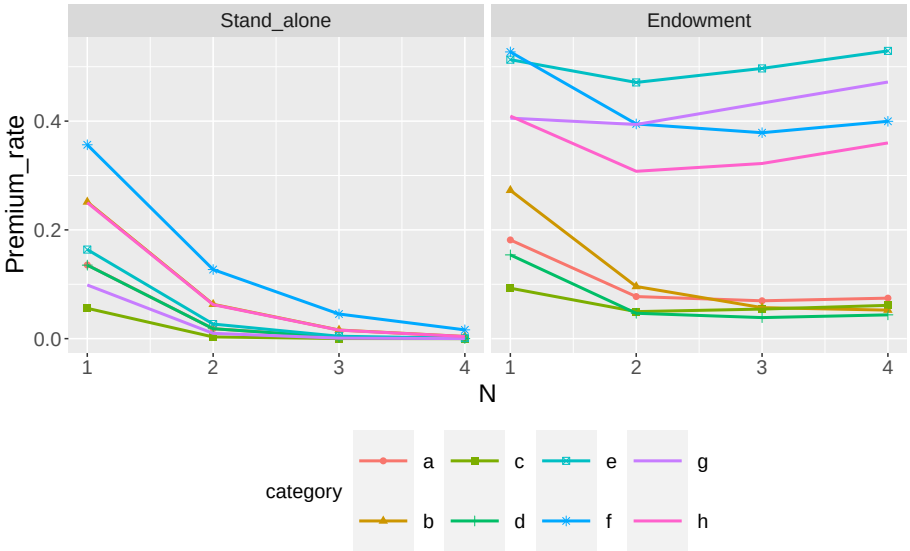

Figure 3: Breast cancer insurance premium at various N.

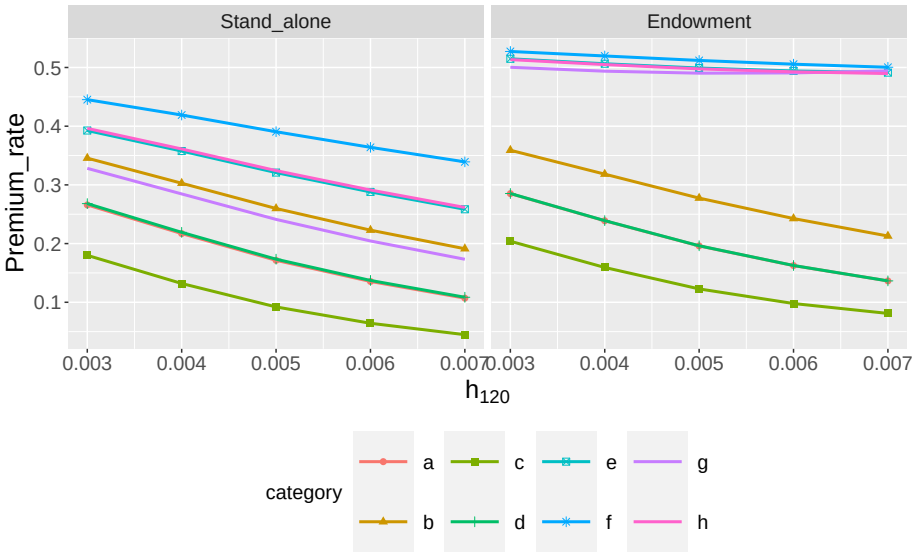

Figure 4: Breast cancer insurance premium at various h_{120}.

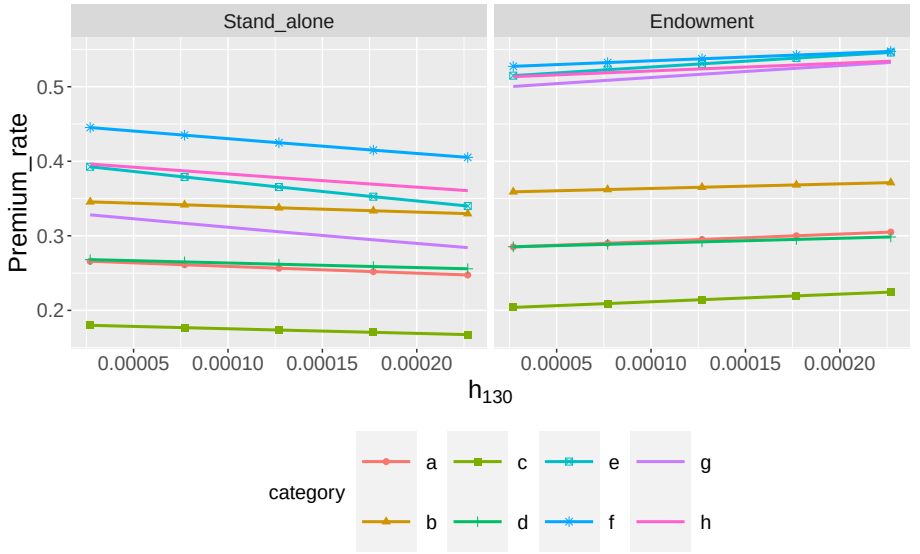

Figure 5: Breast cancer insurance premium at various h_{130}.

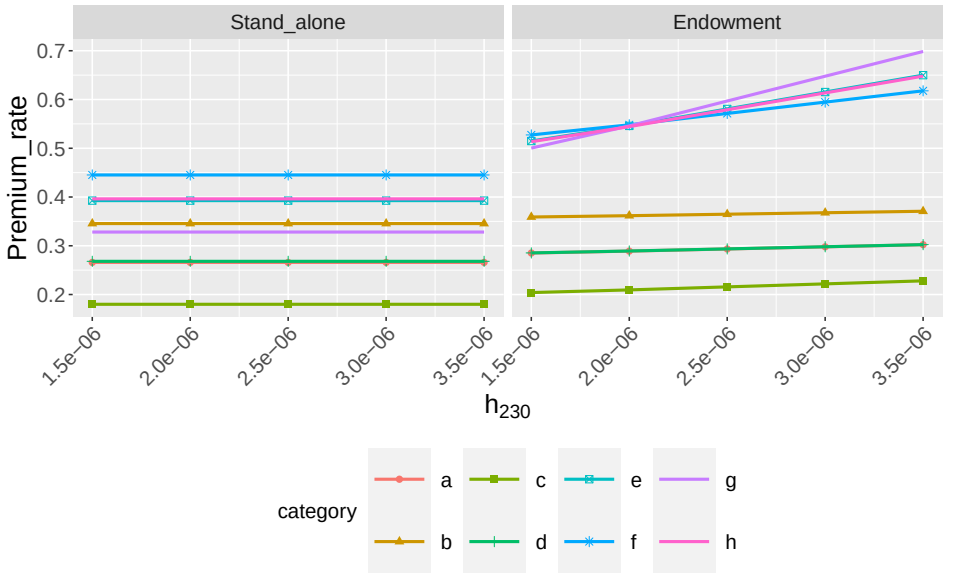

Figure 6: Breast cancer insurance premium at various h_{230}.

Equations (1) and (2) indicate that as t increases, the probability of a breast cancer patient not completing the chemotherapy series decreases, while the probability of the patient passing away rises. Therefore, the duration of the insurance policy contract influences the premium rate. Figure 3 shows that for a stand-alone insurance policy, the premium decreases as the contract length (N) increases. However, the pattern of the impact of contract length on endowment policy premiums is less clearly observable.

A data simulation was conducted to validate the impact of baseline hazard (h_{ij0}) on breast cancer insurance premiums. An increase in the baseline hazard for the transition from state-1 to

state-2 results in a reduction in both stand-alone and endowment insurance premiums (Figure 4). Additionally, the stand-alone premium decreases if the baseline hazard for the transition from state-1 to state-3 rises. Conversely, the endowment insurance premium tends to increase as h_{130} rises (Figure 5). Figure 6 indicates that the baseline hazard for the transition from state-2 to state-3 has no effect on the stand-alone insurance premium; however, in the case of endowment insurance, an increase in h_{230} leads to a higher premium.

5 Discussion and Concluding Remarks

We developed cancer insurance schemes based on the possible transition of the status of breast cancer patients. The Cox model was used to assess the effect of multiple independent variables (covariates) on patient status transitions and estimate transition intensities between patient statuses. According to the multiple-state model with survival framework analysis, cancer stage, age and hypertension status are the three main factors influencing the change in the status of cancer patients. According to the dataset we analyzed, the cancer stage and patient age influence the likelihood of breast cancer patients transitioning to recovery or death within a year. Conversely, comorbidities like hypertension only impact the probability of a status change to death in breast cancer patients after they have completed the chemotherapy series.

The developed model represents a valuable tool for insurance companies to estimate the probabilities of treatment outcomes in breast cancer patients. Within one year of a breast cancer diagnosis, every patient faces the possibility of completing chemotherapy or dying-either before or after treatment completion. Patients in advanced stages with hypertension are more likely to die than those without hypertension. The risk of a patient remaining in an incomplete chemotherapy state one year after diagnosis increases with age and cancer stage. However, patients with hypertension have a higher risk compared to those without hypertension.

This model also offers a beneficial method to estimate critical illness insurance premiums for breast cancer patients, whether through stand-alone benefit policies or endowment benefit policies. For both the stand-alone benefit model and the endowment benefit model, insurance premiums are higher for late-stage and elderly patients compared to earlier-stage and younger patients. Additionally, in both insurance models, patients with comorbidities such as hypertension are obligated to pay higher premiums than those without hypertension.

This study concluded that the duration of the insurance contract significantly impacts premium pricing. In stand-alone insurance policies, longer contract durations tend to yield lower premiums because the probability of breast cancer patients remaining in the status of not having completed chemotherapy diminishes. However, the relationship between contract length and premiums is less distinct in endowment insurance policies. This is attributed to the fact that, while the probability of patients failing to complete the chemotherapy series is relatively low, the risk of mortality remains high. In practice, most critical illness insurance schemes feature short contract durations, typically lasting one year, after which they may be renewed with a new, modified contract. Furthermore, the baseline hazard also plays a crucial role in determining insurance premiums. In stand-alone policies, a higher baseline hazard for the transition from breast cancer diagnosis to chemotherapy completion and subsequent mortality correlates with lower insurance premiums. Conversely, in endowment insurance policies, an increase in the baseline hazard for the transition from diagnosis and chemotherapy completion to the mortality status of patients leads to higher premiums.

The clinical implications of this study's findings are valuable not only for insurance providers but also for doctors and patients. Beyond informing the calculation of insurance premiums, the multi-state model based on Cox analysis provides important information regarding patient risk and prognosis. Doctors can adjust treatment plans to be more personalized and targeted, considering factors such as the patient's age, cancer stage and comorbidity status. For patients, the model aids in better planning for long-term medical expenses. Those at higher risk of status transitions can take a more active role in selecting appropriate insurance policies.

This study has not measured the effects of confounding factors within the model, such as treatment side effects, other health conditions and various elements that could influence the results and risk estimates. Additionally, the study assumes that transition intensities remain constant over time, which may not be valid for other case studies and populations; therefore, a non-homogeneous time assumption may be necessary. Furthermore, the availability of data for this research is quite limited. Consequently, expanding the research population is essential to improve the reliability of the findings. It is important to consider the limitations outlined in this study to ensure transparency and enhance the quality of future research.

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